LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2013**

MT 1819 - PROBABILITY THEORY & STOCHASTIC PROCESSES

Date : 16/11/2013 Dept. No. Max. : 100 Marks
Time : 1:00 - 4:00

Section A

Answer all questions. **(2x10=20)**

1. Define probability space and give an example.
2. What is meant by equally likely events?
3. Define distribution function of a two dimensional random variable.
4. A continuous random variable follows the probability law: $f(x) = a x^2$, $0 \le x \le 1$, determine a.
5. Find the expectation of X, if X is an exponential random variable.
6. Define characteristic function of a random variable.
7. Define convergence in probability.
8. Define a consistent estimator.
9. When is a Markov chain said to be irreducible?
10. How do you find the period of a state in a stochastic process?

Section B

Answer any FIVE questions. **(8x5=40)**

11. Find the moment generating function of exponential distribution and hence find its mean and variance.
12. The contents of urns 1,2 and 3 are as follows:

    Urn 1: 1 white, 2 black and 3 red balls

    Urn 2: 2 white, 1 black and 1 red balls

    Urn 3: 4 white, 5 black and 3 red balls

    One urn is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from urns 1, 2 or 3?
13. Find the first four moments for the following probability density function(pdf):

    $f(x) = 6x(1-x)$, $0 \le x \le 1$.
14. Define convergence in r^{th} mean and show that convergence in r^{th} mean implies convergence in probability.
15. State and prove the invariance property of consistent estimators.

16. Let $x_1, x_2, \ldots x_n$ be a random sample from $N(\mu, \sigma^2)$ population. Find sufficient estimators for μ and σ^2.

17. Find the maximum likelihood estimator for the parameter λ of a Poisson distribution on the basis of a sample size n.

18. A Markov chain on states {1, 2, 3, 4, 5, 6} has transition probability matrix P. Find all equivalence classes and period of states. Also check for the recurrence of the states.

$$P = \begin{bmatrix} 0.5 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0.2 & 0.2 & 0.5 & 0.1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0.4 & 0 & 0 \\ 0.3 & 0 & 0 & 0 & 0.7 & 0 \\ 0.2 & 0.2 & 0.1 & 0 & 0.2 & 0.3 \end{bmatrix}$$

Section C

Answer any TWO questions. **(20x2=40)**

19. (a) The time one has to wait for a bus at a bus stop is observed to be a random phenomenon X with the following pdf:

$$f(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{9}(x+1), & 0 \le x < 1 \\ \frac{4}{9}(x - \frac{1}{2}), & 1 \le x < \frac{3}{2} \\ \frac{4}{9}(\frac{5}{2} - x), & \frac{3}{2} \le x < 2 \\ \frac{1}{9}(4 - x), & 2 \le x \le 3 \\ \frac{1}{9}, & 3 \le x < 6 \\ 0, & x \ge 6 \end{cases}$$

Let the events A and B be defined as follows:

A: One waits between 0 to 2 minutes inclusive,

B: One waits between 1 to 3 minutes inclusive.

(i) Show that f(x) is a pdf.

(ii) Find P(B | A)

(b) State and prove Baye's theorem. (12+8)

20. (a) The joint p.d.f. of two random variables X and Y is given by:

$$\frac{9(1+x+y)}{2(1+x)^4(1+y)^4};\ 0 \le x, y \le \infty$$

Find the marginal distribution of X and Y, and the conditional distribution of Y for X= x.

(b) State and prove Chebyshev's inequality. (10 + 10)

21. (i) Define statistical hypothesis? Explain the concepts of type 1, type 2 errors, power of the test and critical region.

(ii) State and prove Neyman-Pearson lemma. (10 + 10)

22. Explain the postulates of a Poisson process, and derive the transition distribution of the process.

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